

High-temperature Aharonov-Bohm-Casher interferometer

P. M. Shmakov¹, A. P. Dmitriev^{1,2}, and V. Yu. Kachorovskii¹

¹*A.F.Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia*

²*Institut für Nanotechnologie, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*

(Dated: November 1, 2011)

We study theoretically the combined effect of the spin-orbit and Zeeman interactions on the tunneling electron transport through a single-channel quantum ring threaded by magnetic flux. We focus on the high temperature case (temperature is much higher than the level spacing in the ring) and demonstrate that spin-interference effects are not suppressed by thermal averaging. In the absence of the Zeeman coupling the high-temperature tunneling conductance of the ring exhibits two types of oscillations: Aharonov-Bohm oscillations with magnetic flux and Aharonov-Casher oscillations with the strength of the spin-orbit interaction. For weak tunneling coupling both oscillations have the form of sharp periodic antiresonances. In the vicinity of the antiresonances the tunneling electrons acquire spin polarization, so that the ring serves as a spin polarizer. We also demonstrate that the Zeeman coupling leads to appearance of two additional peaks both in the tunneling conductance and in the spin polarization.

PACS numbers: 05.60.+w, 73.40.-c, 73.43.Qt, 73.50.Jt

I. INTRODUCTION

Quantum interferometers based on low-dimensional electronic nanosystems proved to be very powerful tools in studying coherent mesoscopic phenomena¹⁻⁴. The simplest example of such an interferometer is a *single-channel ballistic* quantum ring tunnel-coupled to leads and threaded by the magnetic flux Φ (see Fig. 1). The dependence of the conductance of this setup G on Φ encodes important information about the phase coherence of the tunneling electrons. In particular, the interference of electron waves, propagating in the ring clockwise and counter-clockwise, results in the Aharonov-Bohm (AB) oscillations^{5,6} of G with Φ . The oscillation period is given by the flux quantum $\Phi_0 = hc/e$. The

For $T \ll \Delta$ and weak tunneling coupling there are narrow resonant peaks in the dependence $G(\Phi)$.⁷ The positions of the peaks depend on the electron Fermi energy⁷ and the strength of the electron-electron interaction.⁸ Remarkably, the interference effects are not entirely suppressed by thermal averaging, and the resonant behavior of $G(\Phi)$ survives for the case $T \gg \Delta$. Specifically, the high-temperature conductance of the noninteracting ring with weak tunnel coupling to the contacts exhibits sharp antiresonances at $\phi = 1/2 + N$, where $\phi = \Phi/\Phi_0$ is the dimensionless flux and N is an arbitrary integer number (see Fig.2).^{9,10} The electron-electron interaction leads to appearance of a fine structure of the antiresonances: each antiresonance splits into a series of narrow peaks, whose widths are governed by dephasing.¹⁰

The question which we address in this paper is the role of the spin-orbit (SO) and Zeeman interaction in the high-temperature tunneling transport through the *ballistic single-channel* ring with noninteracting electrons.

The effect of the SO interaction on the properties of one-dimensional (1D) and quasi one-dimensional systems, in particular 1D quantum wires and rings, has attracted much attention.¹¹⁻³⁵ It is known that in a 1D noninteracting wire with an arbitrary spatial dependence of the spin-orbit coupling, the spin degree of freedom may be excluded by a unitary transformation.¹⁸ However, this is not the case for a multiply-connected 1D system such as a single-channel quantum ring. Though such a

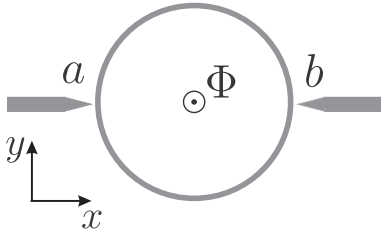


FIG. 1: The ring threaded by magnetic flux Φ .

shape and amplitude of the AB oscillations depend essentially on the strength of the tunneling coupling and on the relation between the temperature T and the level spacing in the ring Δ .

ring is actually a 1D system, the interference between spin parts of two counter-propagating electron waves makes the problem less trivial. Indeed, the rotation of electron spin in the built-in SO magnetic field results in a spin phase shift between clockwise and counterclockwise waves, which is a manifestation of the Aharonov-Casher (AC) effect.^{37,38} The AC phase is the spin analog of the orbital AB phase. More precisely, the AC phase is additional with respect to AB phase and exists even at zero external magnetic field ($\Phi = 0$). An important consequence is the existence of the AC oscillations of zero-field conductance $G(0)$ with the strength of the SO coupling. The AC oscillations were intensively discussed theoretically^{12–17,21,24–27,29–33,35} and their signatures were observed experimentally.^{22,23} Another consequence, especially important from the point of view of possible applications, is that the unpolarized incoming electron beam acquires polarization after passing through the ring, so that the ring may serve as a spin polarizer. The latter effect was recently discussed in a number of publications^{15–17,24,26,27,29,32,33} mostly concerned with the study of the zero-temperature case. The finite temperature effects were also analyzed on the basis of numerical simulations.^{17,26,32}

Here, we develop an analytical theory of the spin-dependent transport through a ballistic single-channel ring with two symmetric contacts focusing on the high temperature case, $T \gg \Delta$. It will be shown that the spin-selective properties of the discussed setup survive thermal averaging. We will see that the SO interaction (in the absence of the Zeeman coupling) leads to the splitting of the high-temperature conductance antiresonances (Fig. 2) into two dips (Fig. 4a), the distance between dips being proportional to the AC phase. In the vicinities of the dips the tunneling electrons acquire polarization $\mathbf{P}(\phi)$ [see Fig. 4b, Eqs. (33) and (42)]. The vector $\mathbf{P}(\phi)$ lies in the (x, z) plane formed by two axes: the axis x , which connects contacts a and b and the axial symmetry axis of the ring (z axis perpendicular to the ring's plane). At zero external field ($\phi = 0$) the conductance $G(0)$ exhibits series of sharp AC antiresonances [see Eq. (44)]. We also demonstrate that taking into account the Zeeman interaction leads to two effects (see Fig. 5): (i) emergence of additional antiresonances in $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$; (ii) appearance of nonzero polarization in y direction.

II. RING WITH SPINLESS ELECTRONS

We start with a brief discussion (see also Ref. 10) of the high-temperature conductance of the ring with spinless electrons. The purpose of this section is to introduce methods, which will be later generalized for the spinful case.

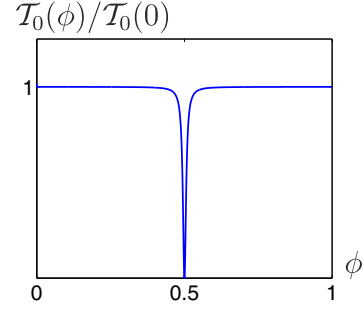


FIG. 2: Dependence of the high-temperature transmission coefficient on the magnetic flux in the spinless case for a ring with weak tunneling coupling to the leads.

The conductance is calculated using the Landauer formula

$$G(\phi) = \frac{e^2}{2\pi\hbar} \mathcal{T}_0(\phi), \quad (1)$$

where

$$\mathcal{T}_0(\phi) = \langle \mathcal{T}_0(\phi, E) \rangle_E = - \int \mathcal{T}_0(\phi, E) \frac{\partial f}{\partial E} dE, \quad (2)$$

is the energy-averaged value of the transmission coefficient $\mathcal{T}_0(\phi, E)$ and $f(E)$ is the Fermi-Dirac function.

We consider both contacts to be identical and describe them with the following S -matrix:

$$S = \begin{bmatrix} t_r & t_{out} & t_{out} \\ t & t_b & t_{in} \\ t & t_{in} & t_b \end{bmatrix}, \quad (3)$$

which relates the amplitudes of three outgoing waves in the channels $(1', 2', 3')$ to the ones in the three incoming channels $(1, 2, 3)$ (see Fig. 3).

In the simplest case of a point tunneling contact one can express all the elements of the S matrix in terms of single real parameter γ :⁷

$$\begin{aligned} t_{in} &= \frac{1}{1 + \gamma}, \quad t_b = -\frac{\gamma}{1 + \gamma}, \\ t = t_{out} &= \frac{\sqrt{2\gamma}}{1 + \gamma}, \quad t_r = -\frac{1 - \gamma}{1 + \gamma}. \end{aligned} \quad (4)$$

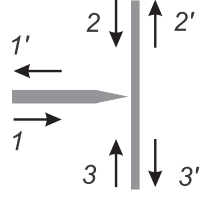


FIG. 3: The scattering on contacts: the amplitude t corresponds to processes $1 \rightarrow 2'$ and $1 \rightarrow 3'$, t_{out} - to $2 \rightarrow 1'$ and $3 \rightarrow 1'$, t_r - to $1 \rightarrow 1'$, t_{in} - to $2 \rightarrow 3'$ and $3 \rightarrow 2'$, t_b - to $2 \rightarrow 2'$ and $3 \rightarrow 3'$.

The case of weak tunneling coupling corresponds to $\gamma \ll 1$. The point metallic-like contact is described by $\gamma \sim 1$.

To find the transmission coefficient we calculate the sum of the amplitudes of all the trajectories that correspond to electron passing through the ring from contact a to contact b . The summation of the amplitudes will be performed in the following way. Each of the trajectories consists of the odd number $2n + 1$ of semicircles connecting the contacts. The length of the trajectory is given by $L_n = \pi R(2n + 1)$, where R is the radius of the ring. Let us denote the sum of the amplitudes of all trajectories with a given length L_n (including trajectories with different number of backscatterings by contacts) as $\beta_n \exp(ikL_n)$, where $k = \sqrt{2mE}/\hbar$ is the electron wave vector. The total transmission amplitude reads

$$t_0(\phi, E) = \sum_{n=0}^{\infty} \beta_n \exp(ikL_n), \quad (5)$$

so that the transmission coefficient is given by

$$\mathcal{T}_0(\phi, E) = |t_0(\phi, E)|^2 = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \beta_n \beta_m^* e^{ik(L_n - L_m)}. \quad (6)$$

We notice now that due to the condition $T \gg \Delta$ the terms in Eq. (6) corresponding to $n \neq m$ vanish after the averaging over E within the temperature window. Hence, the expression for the averaged transmission coefficient becomes

$$\mathcal{T}_0(\phi) = \sum_{n=0}^{\infty} |\beta_n|^2. \quad (7)$$

Next, we write $\beta_n = \beta_n^+ + \beta_n^-$, where β_n^+ (β_n^-) corresponds to trajectories ending with lower (upper) semicircle. It is easy to write the recurrence equations for β_n^+ and β_n^- :

$$\begin{bmatrix} \beta_{n+1}^+ \\ \beta_{n+1}^- \end{bmatrix} = \hat{A}_0 \begin{bmatrix} \beta_n^+ \\ \beta_n^- \end{bmatrix}, \quad (8)$$

where the matrix $\hat{A}_0 = \hat{A}_0(\phi)$ is given by

$$\hat{A}_0 = \begin{bmatrix} t_{in}^2 e^{-2\pi i \phi} + t_b^2 & t_b t_{in} (e^{-2\pi i \phi} + 1) \\ t_b t_{in} (e^{2\pi i \phi} + 1) & t_{in}^2 e^{2\pi i \phi} + t_b^2 \end{bmatrix}. \quad (9)$$

Physically, the elements of matrix \hat{A}_0 are the amplitudes of four different trajectories of length $2\pi R$, starting and ending on the contact b .

From Eqs. (7) and (8) we find

$$\begin{aligned} \mathcal{T}_0(\phi) &= \sum_{n=0}^{\infty} \left| \begin{bmatrix} 1 \\ 1 \end{bmatrix}^\dagger \hat{A}_0^n \begin{bmatrix} \beta_0^+ \\ \beta_0^- \end{bmatrix} \right|^2 \\ &= \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} \beta_0^+ \\ \beta_0^- \end{bmatrix} \right)^\dagger \frac{1}{1 - \hat{A}_0 \otimes \hat{A}_0} \begin{bmatrix} \beta_0^+ \\ \beta_0^- \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \end{aligned} \quad (10)$$

where \otimes denotes the direct (Kronecker) product of matrices and

$$\beta_0^+ = t t_{out} e^{-i\pi\phi}, \quad \beta_0^- = t t_{out} e^{i\pi\phi} \quad (11)$$

represent the amplitudes of shortest counterclockwise and clockwise trajectories, respectively.

Using Eqs. (4), (9), (10), and (11) after some algebra we get the following expression for the transmission coefficient:¹⁰

$$\mathcal{T}_0(\phi) = \frac{2\gamma \cos^2 \pi \phi}{\gamma^2 + \cos^2 \pi \phi}. \quad (12)$$

In the almost closed ring with weak tunneling coupling, $\gamma \ll 1$, Eq. (12) can be well-approximated with the function

$$\mathcal{T}_0(\phi) = \frac{2\gamma \pi^2 (\phi - 1/2)^2}{\gamma^2 + \pi^2 (\phi - 1/2)^2} \quad (13)$$

[Eq.(13) is valid for $0 < \phi < 1$]. This dependence is shown in Fig. 2. We see that there is a sharp antiresonance at $\phi = 1/2$ and $\mathcal{T}_0(1/2) = 0$. The physics behind this behavior can be explained as follows.⁷ For each trajectory there exists a corresponding mirrored (with respect to the x -axis) trajectory. The sum of the amplitudes of these two trajectories is proportional to $e^{ikL_n} (e^{i(2|m|+1)\pi\phi} + e^{-i(2|m|+1)\pi\phi})$, where m is a difference between the number of full clockwise and counterclockwise revolutions, $|m| \leq n$. At $\phi = 1/2$ this sum turns to zero for any k . Thus, the destructive interference at $\phi = 1/2$ survives the thermal averaging.

III. RING WITH SPINFUL ELECTRONS

In the spinful case the Hamiltonian is given by

$$\hat{H} = \hat{H}_{kin} + \hat{H}_Z + \hat{H}_{SO}, \quad (14)$$

where

$$\hat{H}_{kin} = -\frac{\hbar^2}{2mR^2}D_\varphi^2, \quad (15)$$

is the kinetic energy, $D_\varphi = \partial/\partial\varphi + i\phi$,

$$\hat{H}_Z = \frac{1}{2}\hbar\omega_Z\hat{\sigma}_z, \quad (16)$$

is the Zeeman term, $\hbar\omega_Z$ is the Zeeman splitting energy in the external field \mathbf{B} parallel to z axis, and \hat{H}_{SO} corresponds to the SO coupling.

We assume that the SO interaction is described by the Rashba Hamiltonian, which for the case of a straight wire looks $\hat{H}_{SO} = \alpha[\mathbf{n} \times \hat{\boldsymbol{\sigma}}] \cdot \mathbf{p}$. Here \mathbf{n} is the unit vector parallel to built-in electric field, $\hat{\boldsymbol{\sigma}}$ is the vector of the Pauli matrices, α is the constant of the SO interaction, and \mathbf{p} is the electron momentum. In a curved wire, \mathbf{n} depends on the coordinate, and the Hamiltonian becomes^{13,14,18}

$$\hat{H}_{SO} = (\alpha/2)\{[\mathbf{n} \times \hat{\boldsymbol{\sigma}}], \mathbf{p}\}, \quad (17)$$

where $\{\dots\}$ stands for the anticommutator. For a ring with axially symmetric built-in field, $\mathbf{n} = (\cos\varphi \cos\theta, \sin\varphi \cos\theta, \sin\theta)$, we find from Eq. (17)

$$\hat{H}_{SO} = -i\xi \frac{\hbar^2}{2mR^2} \left\{ \begin{bmatrix} -\cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & \cos\theta \end{bmatrix}, D_\varphi \right\}. \quad (18)$$

Here φ is the angle coordinate of the electron in the ring, θ is the angle between effective SO-induced magnetic field \mathbf{B}_{eff} (this field is proportional to $\alpha[\mathbf{p} \times \mathbf{n}]$) and the z axis. The coefficient $\xi = \alpha m R / \hbar$ entering Eq. (18) is the dimensionless parameter characterizing the strength of SO interaction. Physically, ξ is the angle of spin rotation in the local field \mathbf{B}_{eff} during the time on the order of R/v_F , where v_F is the Fermi velocity. In the simplest case $\theta = 0$, ξ is proportional to the angle of the spin rotation after passing around the ring [see Eqs.(29) and (33)].

We study the problem quasiclassically assuming that $k_F R \gg 1$ and $\alpha \ll v_F$ (or, equivalently, $\xi \ll k_F R$). Within this approximation the combined effect of the SO and Zeeman interaction is fully described by the rotation of the electron spin in the field $\mathbf{B} + \mathbf{B}_{\text{eff}}$, which varies along the electron trajectory.^{13,14} Using Eqs. (14)-(18) we find (see Appendix A) the matrices of the spin rotation

$$\begin{aligned} \hat{M}_{a \rightarrow b}^\pm &= \cos \pi \delta_\pm e^{i\vartheta_\pm \hat{\sigma}_y} \pm i \sin \pi \delta_\pm \hat{\sigma}_z, \\ \hat{M}_{b \rightarrow a}^\pm &= (\hat{M}_{a \rightarrow b}^\pm)^T. \end{aligned} \quad (19)$$

Here $\hat{M}_{i \rightarrow j}^+$ ($\hat{M}_{i \rightarrow j}^-$) describes spin rotation for an electron passing a semicircle from contact i to contact j with zero winding number in counter-clockwise (clockwise) directions, \hat{M}^T denotes the transpose of a matrix \hat{M} , and

$$\begin{aligned} \delta_\pm &= |\kappa_\pm| - \frac{1}{2}, \quad e^{i\vartheta_\pm} = \frac{\kappa_\pm}{|\kappa_\pm|}, \\ \kappa_\pm &= \frac{1}{2} + \xi e^{i\theta} \mp \Omega_Z. \end{aligned} \quad (20)$$

The strength of the Zeeman coupling is characterized by dimensionless parameter $\Omega_Z = \omega_Z R / 2v_F$.³⁶

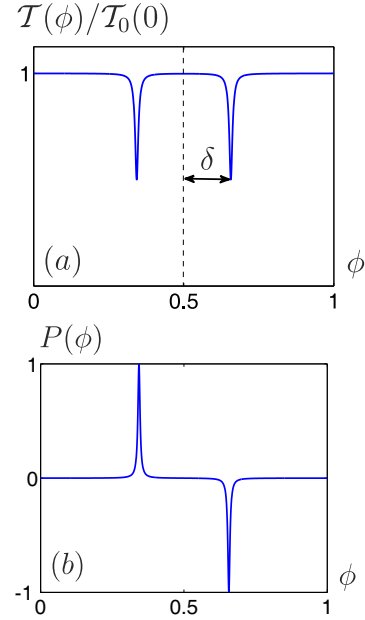


FIG. 4: Transmission coefficient (a) and the spin polarization of the transmitted electrons (b) in the direction of vector $\boldsymbol{\rho}$ for the ring with the SO interaction ($\gamma = 0.02$, $\xi = 0.2$, $\theta = \pi/4$, $\Omega_Z = 0$).

The sum of the amplitudes of the trajectories having length L_n , initial spin state $|\chi_i\rangle$, and final spin state $|\chi_f\rangle$ is given by $\langle\chi_f|\hat{\beta}_n|\chi_i\rangle$, where $\hat{\beta}_n$ are now 2×2 matrices. The amplitude of transmission through the ring with spin state changing from $|\chi_i\rangle$ to $|\chi_f\rangle$ is given by $\langle\chi_i|\hat{t}|\chi_f\rangle$, where

$$\hat{t}(\phi, E) = \sum_{n=0}^{\infty} \hat{\beta}_n \exp(ikL_n). \quad (21)$$

The transmission coefficient reads

$$\mathcal{T} = \frac{1}{2} \langle \text{Tr } \hat{t} \hat{t}^\dagger \rangle_E = \frac{1}{2} \text{Tr } \hat{\mathcal{T}}, \quad (22)$$

where

$$\hat{\mathcal{T}} = \sum_{n=0}^{\infty} \hat{\beta}_n \hat{\beta}_n^\dagger. \quad (23)$$

The electrons passing through the ring acquire spin polarization. For the case of unpolarized incoming electron beam the spin polarization is calculated as

$$\mathbf{P} = \frac{\langle \text{Tr } \hat{\sigma} \hat{t} \hat{t}^\dagger \rangle_E}{2\mathcal{T}} = \frac{\text{Tr } \hat{\sigma} \hat{\mathcal{T}}}{2\mathcal{T}}, \quad (24)$$

and, therefore, is also expressed in terms of $\hat{\beta}_n$.

To find $\hat{\beta}_n$ we separate trajectories into two groups (just as in the previous section), writing $\hat{\beta}_n = \hat{\beta}_n^+ + \hat{\beta}_n^-$, where $\hat{\beta}_n^+$ and $\hat{\beta}_n^-$ satisfy the recurrence equations, analogous to Eq.(8):

$$\begin{bmatrix} \hat{\beta}_{n+1}^+ \\ \hat{\beta}_{n+1}^- \end{bmatrix} = \hat{A} \begin{bmatrix} \hat{\beta}_n^+ \\ \hat{\beta}_n^- \end{bmatrix}. \quad (25)$$

The block matrix \hat{A} is given by

$$\hat{A} = \begin{bmatrix} e^{-2\pi i \phi} t_{in}^2 \hat{M}_+ + t_b^2 \hat{K} & t_b t_{in} (e^{-2\pi i \phi} \hat{M}_+ + \hat{K}) \\ t_b t_{in} (e^{2\pi i \phi} \hat{M}_- + \hat{K}') & e^{2\pi i \phi} t_{in}^2 \hat{M}_- + t_b^2 \hat{K}' \end{bmatrix}.$$

Here $\hat{M}_+ = \hat{M}_{a \rightarrow b}^+ \hat{M}_{b \rightarrow a}^+$, $\hat{M}_- = \hat{M}_{a \rightarrow b}^- \hat{M}_{b \rightarrow a}^-$, $\hat{K} = \hat{M}_{a \rightarrow b}^+ \hat{M}_{b \rightarrow a}^-$, $\hat{K}' = \hat{M}_{a \rightarrow b}^- \hat{M}_{b \rightarrow a}^+$. The matrix \hat{M}_+ (\hat{M}_-) describes spin rotation after passing a full circle starting from contact b and propagating in counterclockwise (clockwise) direction. The matrix \hat{K} (\hat{K}') is the spin rotation matrix for an electron, which starts from the contact b , then propagates along the lower (upper) shoulder of the interferometer and returns back to the contact b after single backscattering on the contact a .

For the shortest trajectories we have

$$\begin{aligned} \hat{\beta}_0^+ &= t t_{out} e^{-i\pi\phi} \hat{M}_{a \rightarrow b}^+, \\ \hat{\beta}_0^- &= t t_{out} e^{i\pi\phi} \hat{M}_{a \rightarrow b}^-. \end{aligned} \quad (26)$$

The matrix elements of matrix $\hat{\mathcal{T}}$ can be written in a form, analogous to Eq.(10):

$$\begin{aligned} \langle \chi_f | \hat{\mathcal{T}} | \chi_i \rangle &= \sum_{n,k} \langle \chi_f | \hat{\beta}_n | \chi_k \rangle \langle \chi_k | \hat{\beta}_n^\dagger | \chi_i \rangle \\ &= \sum_k \begin{bmatrix} \chi_f \\ \chi_f \end{bmatrix}^\dagger \otimes \begin{bmatrix} \hat{\beta}_0^+ \chi_k \\ \hat{\beta}_0^- \chi_k \end{bmatrix}^\dagger \frac{1}{1 - \hat{A} \otimes \hat{A}^\dagger} \begin{bmatrix} \hat{\beta}_0^+ \chi_k \\ \hat{\beta}_0^- \chi_k \end{bmatrix} \otimes \begin{bmatrix} \chi_i \\ \chi_i \end{bmatrix}. \end{aligned} \quad (27)$$

In the following sections we will use Eqs. (19)-(27) to calculate the full transmission coefficient and the spin polarization for a ring with the SO and Zeeman interactions.

A. Ring with SO interaction (zero Zeeman coupling)

In the absence of the Zeeman coupling ($\Omega_Z = 0$) we find from Eq. (20)

$$\delta_+ = \delta_- = \delta, \quad \vartheta_+ = \vartheta_- = \vartheta. \quad (28)$$

Here

$$\delta = \sqrt{\frac{1}{4} + \xi \cos \theta + \xi^2} - \frac{1}{2}, \quad (29)$$

$$\tan \vartheta = \frac{\xi \sin \theta}{1/2 + \xi \cos \theta}. \quad (30)$$

Now we will make use of the important feature of SO interaction: if the electron travels along a certain trajectory and then returns to the initial point along the same trajectory moving in the opposite direction, its spin returns to the original state. This implies that

$$\hat{K} = \hat{K}' = 1, \quad \hat{M}_+ = \hat{M}_-^{-1} = \hat{M} \quad (31)$$

[one can easily check Eq. (31) using Eqs. (19) and (28)]. These properties essentially simplify further calculations. The block matrix \hat{A} is now fully expressed in terms of matrix \hat{M}

$$\hat{A} = \begin{bmatrix} t_{in}^2 e^{-2\pi i \phi} \hat{M} + t_b^2 & t_b t_{in} (e^{-2\pi i \phi} \hat{M} + 1) \\ t_b t_{in} (e^{2\pi i \phi} \hat{M}^{-1} + 1) & t_{in}^2 e^{2\pi i \phi} \hat{M}^{-1} + t_b^2 \end{bmatrix}.$$

Next, we write

$$\hat{M} = \exp(-i\boldsymbol{\rho} \hat{\sigma}/2), \quad (32)$$

where $\boldsymbol{\rho}$ is the vector of spin rotation for counterclockwise propagation around the ring (starting from contact b). From Eqs. (19), (28), (29), and (30) we find

$$\boldsymbol{\rho} = 4\pi\delta(e_x \sin \vartheta - e_z \cos \vartheta). \quad (33)$$

The eigenvectors of \hat{M} are the spinors χ^\uparrow and χ^\downarrow corresponding to spin orientation along $\boldsymbol{\rho}$ and $-\boldsymbol{\rho}$:

$$\hat{M} \chi^\uparrow = \exp(-i2\pi|\delta|) \chi^\uparrow, \quad (34)$$

$$\hat{M} \chi^\downarrow = \exp(i2\pi|\delta|) \chi^\downarrow. \quad (35)$$

As follows from Eqs. (34) and (35), $2\pi|\delta|$ is the Aharonov-Casher (AC) phase^{37,38} induced by the SO interaction. Using Eq. (29) we find that δ lies in the interval between $\sqrt{\xi^2 + 1/4} - 1/2$ and ξ . These limiting values are realized, respectively,

for $\theta = \pi/2$ (\mathbf{B}_{eff} is parallel to the ring plane) and $\theta = 0$ (\mathbf{B}_{eff} is parallel to the z axis).

For $\xi \gg 1$, the frequency of spin precession in the field \mathbf{B}_{eff} is much larger than the orbital frequency v_F/R and the direction of the spin follows adiabatically the direction of \mathbf{B}_{eff} . In this case Eq. (29) simplifies

$$2\pi\delta = 2\pi\xi - \pi(1 - \cos\theta), \quad \text{for } \xi \gg 1. \quad (36)$$

Thus, in the adiabatic limit the AC phase separates into two parts:¹³ dynamical contribution $2\pi\xi$ and geometrical SO Berry phase³⁹ $\pi(1 - \cos\theta)$ which is the half of the solid angle subtended by \mathbf{B}_{eff} when electron passes the full circle.⁴⁰

In order to find the recurrence equations for $\hat{\beta}_n^\pm$ we now introduce the spinors $\tilde{\chi}^{\uparrow\downarrow} = \exp(\mp i\pi|\delta|)(\hat{M}_{a \rightarrow b}^\dagger)^{-1}\chi^{\uparrow\downarrow}$, which are transformed to $\chi^{\uparrow\downarrow}$ when the electron propagates from contact a to contact b [the phase multiplier $\exp(\mp i\pi|\delta|)$ is added for convenience]. Using Eq.(25) we get $\langle \chi^\downarrow | \hat{\beta}_n | \tilde{\chi}^\uparrow \rangle = \langle \chi^\uparrow | \hat{\beta}_n | \tilde{\chi}^\downarrow \rangle = 0$, so that $\hat{\beta}_n$ can be written as

$$\hat{\beta}_n = \beta_n^\uparrow |\chi_\uparrow\rangle \langle \tilde{\chi}_\uparrow| + \beta_n^\downarrow |\chi_\downarrow\rangle \langle \tilde{\chi}_\downarrow|. \quad (37)$$

For $\beta_n^\uparrow = \langle \chi^\uparrow | \hat{\beta}_n | \tilde{\chi}^\uparrow \rangle$ we get the recurrence equations

$$\begin{bmatrix} \beta_{n+1}^{\uparrow+} \\ \beta_{n+1}^{\uparrow-} \end{bmatrix} = \hat{A}_0(\phi + |\delta|) \begin{bmatrix} \beta_n^{\uparrow+} \\ \beta_n^{\uparrow-} \end{bmatrix}, \quad (38)$$

Here $\hat{A}_0(\phi)$ is given by Eq. (9) and

$$\begin{bmatrix} \beta_0^{\uparrow+} \\ \beta_0^{\uparrow-} \end{bmatrix} = \begin{bmatrix} e^{-i\pi(\phi+|\delta|)} \\ e^{i\pi(\phi+|\delta|)} \end{bmatrix}. \quad (39)$$

We see that the quantities $\beta_n^{\uparrow\pm}$ satisfy the same recurrence equations as the ones in the spinless case [see Eq. (8)] with the replacement ϕ with $\phi + |\delta|$. One can easily show that the recurrence equations for $\beta_n^\downarrow = \langle \chi^\downarrow | \hat{\beta}_n | \tilde{\chi}^\downarrow \rangle$ are given by Eqs. (38) and (39) with the replacement $\phi + |\delta|$ with $\phi - |\delta|$. As a result, we find

$$\hat{T}(\phi) = \mathcal{T}_0(\phi - |\delta|) |\chi_\uparrow\rangle \langle \tilde{\chi}_\uparrow| + \mathcal{T}_0(\phi + |\delta|) |\chi_\downarrow\rangle \langle \tilde{\chi}_\downarrow|, \quad (40)$$

where \mathcal{T}_0 is the transmission coefficient of the spinless electrons given by Eq.(9). The expressions for the full transmission coefficient and the spin polarization become

$$\mathcal{T}(\phi) = \frac{\mathcal{T}_0(\phi + \delta) + \mathcal{T}_0(\phi - \delta)}{2}, \quad (41)$$

$$\mathbf{P}(\phi) = P(\phi) \frac{\boldsymbol{\rho}}{\rho}, \quad (42)$$

where

$$P(\phi) = \frac{\mathcal{T}_0(\phi + |\delta|) - \mathcal{T}_0(\phi - |\delta|)}{\mathcal{T}_0(\phi + |\delta|) + \mathcal{T}_0(\phi - |\delta|)}. \quad (43)$$

It is worth noting that Eq. (41) is in agreement with the general theorem, relating any transport property of 1D system with the SO interaction with the same property without the SO interaction.¹¹

The dependencies of the conductance and the spin polarization on magnetic flux are schematically depicted in Fig.4. As seen, there are two dips (per period) in the function $\mathcal{T}(\phi)$, corresponding to $\phi = 1/2 \pm \delta + N$. At these two points the incoming electrons with spin states described, respectively, by $\tilde{\chi}^\downarrow$ and $\tilde{\chi}^\uparrow$ are totally blocked by the destructive interference. Therefore, the tunneling current becomes fully polarized in the direction of $\boldsymbol{\rho}$ for $\phi = 1/2 \pm \delta + N$.

We see that SO-induced splitting of the resonances is proportional to the AC phase $2\pi\delta$. Eqs. (41) and (43) reveal coexisting of two types of oscillations: the AB oscillations with ϕ and AC oscillations with δ . Importantly, AC oscillations of tunneling conductance exist even in the case of zero external field. Indeed, for $\phi = 0$, we have

$$\mathcal{T} = \mathcal{T}_0(\delta), \quad P = 0. \quad (44)$$

Here we took into account that $\mathcal{T}_0(\delta)$ is an even function. Thus, transmission coefficient exhibits the AC oscillations with the period $\delta = 1$. For the case of almost closed ring, $\gamma \ll 1$, the oscillations have the form of sharp antiresonances periodic in δ .

In conclusion of this section, we note that Eqs. (41), (43), and (44) are valid for $T \gg \Delta$ and arbitrary strength of tunneling coupling ($0 < \gamma < \infty$). They represent a generalization of the analytical results obtained previously^{15,17,21,24,26} for $T = 0$ and strong tunneling coupling (metallic-like contacts, $\gamma \simeq 1$).

B. Interplay of spin-orbit and Zeeman interactions

Next we discuss the role of the Zeeman interaction. Taking this interaction into account requires much more tricky calculations. The point is that the properties (31) are no longer valid when the time reversal symmetry is broken. Consequently, the elements of block matrix \hat{A} can not

be expressed in terms of a single rotation matrix. In principle, the expressions for \mathcal{T} and \mathbf{P} can be derived from (27), where both averaging over the temperature window and summation over winding number n are already done. This equation, indeed, turns out to be very useful for numerical simulations. However, the analytical expres-

sions obtained with the use of (27) for the case of arbitrary γ turn out to be very cumbersome and we do not present them here. We restrict ourselves with the analytical study of the almost closed ring, $\gamma \ll 1$. For this case, the calculations presented in the Appendix B yield

$$\mathcal{T}(\phi) = \frac{c_-^2 [\mathcal{T}_0(\phi + \delta) + \mathcal{T}_0(\phi - \delta)] + s_-^2 [\mathcal{T}_0(\phi + \delta') + \mathcal{T}_0(\phi - \delta')]}{2}, \quad (45)$$

$$P_x(\phi) = \frac{s_+ c_- [\mathcal{T}_0(\phi + \delta) - \mathcal{T}_0(\phi - \delta)] + s_- c_+ [\mathcal{T}_0(\phi + \delta') - \mathcal{T}_0(\phi - \delta')]}{2\mathcal{T}(\phi)}, \quad (46)$$

$$P_y(\phi) = \frac{s_- c_- [\mathcal{T}'_0(\phi + \delta') + \mathcal{T}'_0(\phi - \delta') - \mathcal{T}'_0(\phi + \delta) - \mathcal{T}'_0(\phi - \delta)]}{2\mathcal{T}(\phi)}, \quad (47)$$

$$P_z(\phi) = \frac{c_+ c_- [\mathcal{T}_0(\phi - \delta) - \mathcal{T}_0(\phi + \delta)] + s_+ s_- [\mathcal{T}_0(\phi + \delta') - \mathcal{T}_0(\phi - \delta')]}{2\mathcal{T}(\phi)}, \quad (48)$$

where

$$\delta = \frac{\delta_+ + \delta_-}{2}, \quad \delta' = \frac{\delta_+ - \delta_-}{2} + \frac{1}{2}, \quad (49)$$

$$s_{\pm} = \sin\left(\frac{\vartheta_{\pm} \pm \vartheta_-}{2}\right), \quad (50)$$

$$c_{\pm} = \cos\left(\frac{\vartheta_{\pm} \pm \vartheta_-}{2}\right), \quad (51)$$

and

$$\mathcal{T}'_0(\phi) = \frac{2\pi\gamma^2(\phi - 1/2)}{\gamma^2 + \pi^2(\phi - 1/2)^2}. \quad (52)$$

These results are shown in Fig. 5. We see that in the presence of the Zeeman interaction instead of the two antiresonances there are four ones (per period) corresponding to the flux values $1/2 \pm \delta + N$ and $1/2 \pm \delta' + N$. In the vicinity of each antiresonance the outgoing electrons are polarized. Importantly, the Zeeman coupling induces nonzero polarization in y direction. One may notice that the dependence $P_y(\phi)$ is qualitatively different from dependencies $P_x(\phi)$ and $P_z(\phi)$. First of all, the peaks in $P_y(\phi)$ are asymmetric [see Eq. (52)] in contrast to the peaks in $P_x(\phi)$ and $P_z(\phi)$. Secondly, all four resonances in $P_y(\phi)$ have the same amplitudes.

Next, we consider some limiting cases. For weak Zeeman coupling, $|\Omega_Z| \ll \max\{1, |\xi|\}$, one

finds

$$\begin{aligned} \delta &\approx \frac{\xi \cos \theta + \xi^2}{\sqrt{1/4 + \xi \cos \theta + \xi^2} + 1/2}, \quad \delta' \approx 1/2 - \Omega_Z, \\ c_- &\approx 1, \quad s_- \approx \frac{\Omega_Z \xi \sin \theta}{1/4 + \xi \cos \theta + \xi^2}, \\ c_+ &\approx \frac{1/2 + \xi \cos \theta}{\sqrt{1/4 + \xi \cos \theta + \xi^2}}, \\ s_+ &\approx \frac{\xi \sin \theta}{\sqrt{1/4 + \xi \cos \theta + \xi^2}}. \end{aligned} \quad (53)$$

In the strong Zeeman coupling limit, $|\Omega_Z| \gg \max\{1, |\xi|\}$, we obtain

$$\begin{aligned} \delta &\approx 1/2 - \Omega_Z, \quad \delta' \approx \xi \cos \theta, \\ c_- &\approx \frac{\xi \sin \theta}{|\Omega_Z|}, \quad s_- \approx \text{sign } \Omega_Z, \\ c_+ &\approx \frac{\xi \sin \theta (\frac{1}{2} + \xi \cos \theta)}{\Omega_Z^2}, \quad s_+ \approx 1. \end{aligned} \quad (54)$$

We see that in both limiting cases there are two deep antiresonances, the positions of which are controlled by the strength of the SO interaction, and two small ones with the positions controlled by the Zeeman coupling.

The competition between spin-orbit and Zeeman coupling is clearly seen in the case $\xi \gg 1$, $\Omega_Z \gg 1$, and arbitrary relation between ξ and Ω_Z . In particular, the amplitudes of peaks

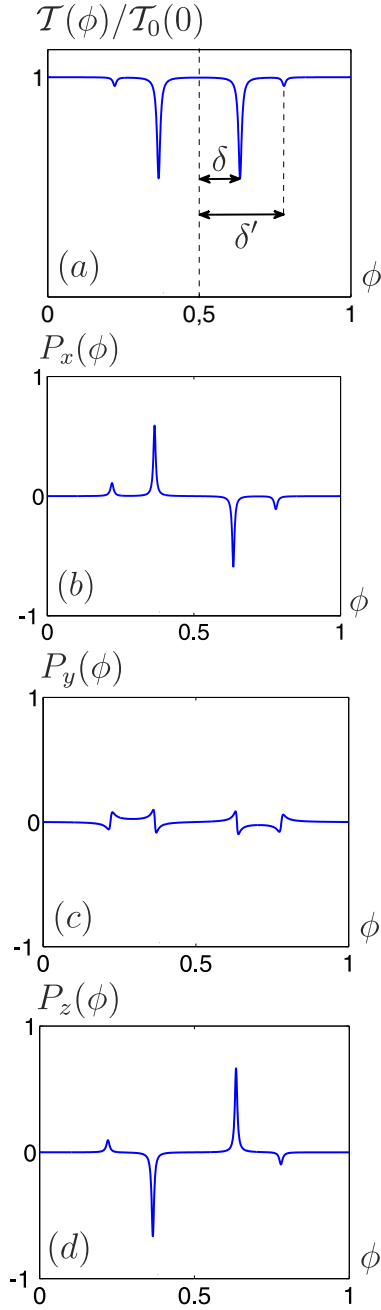


FIG. 5: Transmission coefficient through the ring with the SO and Zeeman interactions (a) and spin polarization of the transmitted electrons in the x , y and z directions (b), (c) and (d), respectively ($\gamma = 0.02$, $\xi = 1.9$, $\theta = \pi/4$, $\Omega_Z = 1$).

in $\mathcal{T}(\phi)$ are given by

$$\frac{s_-^2}{2} \approx \frac{1}{2} \left(1 + \frac{\Omega_Z^2 - \xi^2}{\sqrt{(\Omega_Z^2 - \xi^2)^2 + 4\Omega_Z^2 \xi^2 \sin^2 \theta}} \right),$$

$$\frac{c_-^2}{2} \approx \frac{1}{2} \left(1 - \frac{\Omega_Z^2 - \xi^2}{\sqrt{(\Omega_Z^2 - \xi^2)^2 + 4\Omega_Z^2 \xi^2 \sin^2 \theta}} \right).$$

As follows from these equations, for $\xi \approx \Omega_Z$ all four antiresonances have the same amplitudes.

The dependencies of the positions of four antiresonances on Ω_Z with fixed ξ is shown in Fig. 6. It is noteworthy that in the special case $\theta = \pi/2$, the distance between the deep antiresonances tends to zero when the strength of the Zeeman coupling increases, so that $\mathcal{T}(\phi) \rightarrow \mathcal{T}_0(\phi)$ when $\Omega_Z \rightarrow \infty$. This is also illustrated in Fig. 7 where $\mathcal{T}(\phi)$ is plotted for $\theta = \pi/4$ and $\theta = \pi/2$.

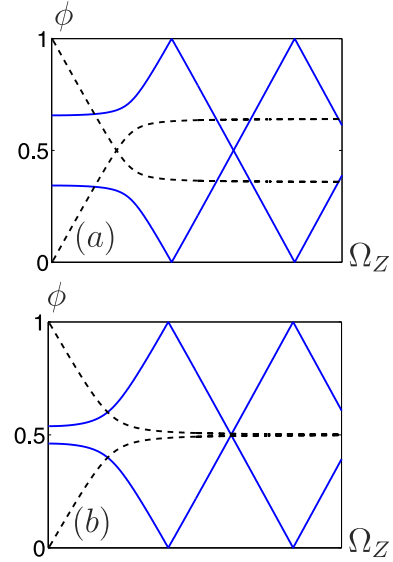


FIG. 6: Positions of antiresonances in the region $0 < \phi < 1$ as functions of Ω_Z for $\xi = 0.2$ and different values of θ : $\theta = \pi/4$ (a) and $\theta = \pi/2$ (b). Solid and dashed lines correspond to $1/2 \pm \delta + N$ and $1/2 \pm \delta' + N$, respectively.

At the end of this section we note that the Zeeman interaction may also lead to inhomogeneous broadening of the antiresonances. Indeed, in the above calculations we replaced in the dimensionless parameter Ω_Z the energy-dependent electron velocity v with v_F . In fact, the positions of antiresonances [see Eq. (49)] depend on v , so that one should average Eqs (45)-(48) over the temperature fluctuations of Ω_Z . Having in

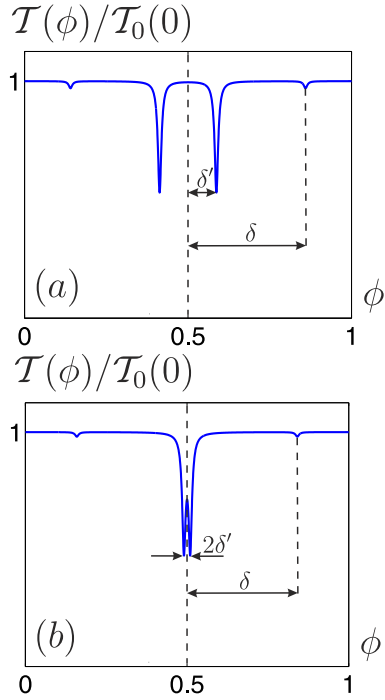


FIG. 7: Transmission coefficient for the strong Zeeman splitting: $\xi = 1.6$, $\Omega_Z = 5$, $\theta = \pi/4$ (a), $\xi = 1.6$, $\Omega_Z = 8$, $\theta = \pi/2$ (b). For $\theta = \pi/2$ the effect of the SO coupling is suppressed with increasing the Zeeman coupling (b).

mind that such fluctuations are on the order of $\Omega_Z T/E_F$, one can conclude that the inhomogeneous broadening should be taken into account when $\Omega_Z T/E_F$ becomes larger than γ .

IV. SUMMARY

In this paper we calculated the high-temperature transmission coefficient $\mathcal{T}(\phi)$ and the spin polarization $\mathbf{P}(\phi)$ of the electrons tunneling through a single-channel ring with the Rashba SO interaction threaded by magnetic flux. We obtained analytical expressions for $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$ valid for arbitrary strength of the tunneling coupling. Both $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$ reveal coexistence of two types of periodic oscillations: the Aharonov-Bohm oscillations with magnetic flux and the Aharonov-Casher oscillations with the strength of SO interaction. For weak tunneling coupling, the oscillations have the form of the sharp antiresonances periodic in ϕ and δ . Specifically, in the absence of the Zeeman coupling there are two antiresonances (per period) in the depen-

dence $\mathcal{T}(\phi)$ (instead of one antiresonance in the spinless case). In the vicinity of each antiresonance, the electron beam passing through the ring acquires strong spin polarization directed in the (x, z) plane formed by axial symmetry axis of the ring (z axis) and the line connecting two contacts. We also discussed the influence of the Zeeman interaction on the interference picture and showed that two additional antiresonances appear both in $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$. Also, the Zeeman coupling leads to appearance of nonzero spin polarization in y direction.

Let us finally briefly discuss some unsolved problems. In the above calculations we assumed that electrons are noninteracting. The detailed analysis of the effects caused by electron-electron interaction is out of scope of the current paper. Here we restrict ourselves to a very brief qualitative discussion (for the case $T \ll \Delta$, the role of the electron-electron interaction in a ring with SO interaction was discussed in Refs. 25). First of all, the interaction leads to a renormalization of the tunneling rate due to Luttinger-liquid correlations specific for purely 1D systems. This implies that in all equations derived above one should replace γ by renormalized tunneling rate $\tilde{\gamma}$ (for discussion of renormalization of 3×3 matrix of the tunneling contact in the spinless case see Refs. 42–46). This does not change our results much, since γ is a phenomenological parameter of the model. What is more important is that the interaction suppresses the interference between clockwise and counterclockwise propagating electron waves, thus leading to a broadening of the resonances. It is expected⁴⁷ that this broadening is negligible when interaction is so weak that

$$\tilde{\gamma}\Delta \gg \Gamma_\varphi^0, \quad (55)$$

where $\Gamma_\varphi^0 \simeq gT$ is the electron-electron scattering rate in the infinite single-channel wire⁴⁸ (here g is dimensionless constant characterizing the strength of the electron-electron interaction) Equations derived in the previous sections are valid (with the replacement of γ with $\tilde{\gamma}$) provided that inequality (55) is satisfied. In the opposite limiting case $\tilde{\gamma}\Delta \ll \Gamma_\varphi^0$, the interaction should modify the structure of the resonant peaks in $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$. One may expect two effects, predicted previously for the spinless case:¹⁰ (i) all the resonances would acquire fine structure, splitting into a series of narrow peaks separated by distance g (ii) each of the peaks of the split structure would broaden within a width Γ_φ/Δ , where Γ_φ is the phase breaking rate, which is

expected to be much smaller than the bulk dephasing rate ($\Gamma_\varphi \ll \Gamma_\varphi^0$) because of the charge and size quantization in the almost closed ring. More detailed analysis including rigorous calculation of Γ_φ will be presented elsewhere.⁴⁷

V. ACKNOWLEDGMENTS

We thank I.V. Gornyi and D.G. Polyakov for valuable discussions. The work was supported by Russian Foundation for Basic Research (grants 11-02-00146 and 11-02-91346) and by programs of the Russian Academy of Science. The work of P.M. Shmakov was supported by Dynasty Foundation.

Appendix A

In this appendix we discuss the derivation of the spin rotation matrices (19). Let us denote as $M(\varphi_0, \varphi)$ the spin rotation matrix, which corresponds to the counterclockwise trajectory with zero winding number going from φ_0 to φ . To find this matrix we write

$$\hat{M}(\varphi_0, \varphi + \delta\varphi) = \delta\hat{M}(\varphi)\hat{M}(\varphi_0, \varphi), \quad (\text{A1})$$

where $\delta\hat{M}(\varphi) = \hat{M}(\varphi, \varphi + \delta\varphi)$ describes the spin rotation along trajectory with an infinitesimally short length.

In the quasiclassical approximation an electron spin state χ obeys the following equation^{13,14}

$$i\hbar \frac{\partial \chi}{\partial \varphi} \frac{v_F}{R} = \hat{H}' \chi, \quad (\text{A2})$$

where \hat{H}' is the Hamiltonian given by the sum of Eqs. (14) and (17), with the operator $-i\partial/\partial\varphi$ in \hat{H}_{SO} replaced with $k_F R$ (for $\varphi > \varphi_0$) or with $-k_F R$ (for $\varphi < \varphi_0$). From Eq. (A2) one can easily conclude that $\delta\hat{M}(\varphi) = 1 - iR\delta\varphi\hat{H}'/\hbar v_F$. As a result, we find that the matrix $\hat{M}(\varphi_0, \varphi)$ obeys the following differential equation

$$\frac{\partial \hat{M}(\varphi_0, \varphi)}{\partial \varphi} = i \begin{bmatrix} l_z & -l_\perp e^{-i\varphi} \\ -l_\perp e^{i\varphi} & -l_z \end{bmatrix} \hat{M}(\varphi_0, \varphi) \quad (\text{A3})$$

with initial condition $\hat{M}(\varphi_0, \varphi_0) = 1$. Here $l_z = \xi \cos \theta - \Omega_Z$ and $l_\perp = \xi \sin \theta$. The substitution

$$\hat{M} = \hat{M}' \begin{bmatrix} e^{-i\varphi/2} & 0 \\ 0 & e^{i\varphi/2} \end{bmatrix}$$

yields

$$\frac{\partial \hat{M}'}{\partial \varphi} = i \begin{bmatrix} l_z + \frac{1}{2} & -l_\perp \\ -l_\perp & -l_z - \frac{1}{2} \end{bmatrix} \hat{M}'. \quad (\text{A4})$$

Solving (A4) we find

$$\hat{M} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad (\text{A5})$$

where

$$\begin{aligned} M_{11} &= e^{i\delta_+(\varphi-\varphi_0)} \cos^2 \frac{\vartheta_+}{2} \\ &\quad + e^{i(1+\delta_+)(\varphi-\varphi_0)} \sin^2 \frac{\vartheta_+}{2}, \\ M_{12} &= -i \sin \vartheta_+ \sin[|\kappa_+|(\varphi - \varphi_0)] e^{-i\frac{\varphi+\varphi_0}{2}}, \\ M_{21} &= -M_{12}^*, \quad M_{22} = M_{11}^*. \end{aligned}$$

Here δ_+ , ϑ_+ and κ_+ are given by Eq. (20).

The case $\varphi < \varphi_0$ is considered in an analogous way, leading to the matrix, given by Eq. (A5), with the replacements $\delta_+ \rightarrow \delta_-$, $\vartheta_+ \rightarrow \vartheta_-$ and $\kappa_+ \rightarrow \kappa_-$. One can check that the matrices $\hat{M}_{a \rightarrow b}^\pm = \hat{M}(\mp\pi, 0)$ are given by Eq.(19) and that $\hat{M}_{b \rightarrow a}^\pm = \hat{M}(0, \pm\pi) = (\hat{M}_{a \rightarrow b}^\pm)^T$.

Appendix B

In this appendix we derive Eqs.(45)-(48) for the transmission coefficient and spin polarization of the electrons, passing through the ring with the Rashba and Zeeman interactions in the limit of weak coupling to the contacts ($\gamma \ll 1$).

For the purpose of this appendix it is convenient to introduce the probability $W_{\chi_i \chi_f} = \langle |\langle \chi_f | \hat{t} | \chi_i \rangle|^2 \rangle_E$ for an electron, which approached the contact a in the spin state $|\chi_i\rangle$, to exit the ring from the contact b in the spin state $|\chi_f\rangle$.

The transmission coefficient and spin polarization are expressed in terms of $W_{\chi_i \chi_f}$ as follows:

$$\mathcal{T} = \frac{W_{\chi_i^\uparrow \chi_f^\uparrow} + W_{\chi_i^\downarrow \chi_f^\uparrow} + W_{\chi_i^\uparrow \chi_f^\downarrow} + W_{\chi_i^\downarrow \chi_f^\downarrow}}{2}, \quad (\text{B1})$$

$$\mathbf{Pn} = \frac{W_{\chi_i^\uparrow \chi_n^\uparrow} + W_{\chi_i^\downarrow \chi_n^\uparrow} - W_{\chi_i^\uparrow \chi_n^\downarrow} - W_{\chi_i^\downarrow \chi_n^\downarrow}}{2\mathcal{T}}, \quad (\text{B2})$$

where $\chi_i^{\uparrow\downarrow}$ and $\chi_f^{\uparrow\downarrow}$ are two arbitrary bases, $\chi_n^{\uparrow\downarrow}$ are the eigenstates of the operator $\hat{\sigma}\mathbf{n}$ ($\hat{\sigma}\mathbf{n}\chi_n^{\uparrow\downarrow} = \pm\chi_n^{\uparrow\downarrow}$) and \mathbf{n} is a unit vector in arbitrary direction.

The probability $W_{\chi_i \chi_f}$ can be found as

$$\begin{aligned} W_{\chi_i \chi_f} &= \sum_{n=0}^{\infty} |\langle \chi_f | \hat{\beta}_n | \chi_i \rangle|^2 \\ &= \sum_{n=0}^{\infty} \left| \begin{bmatrix} \chi_f \\ \chi_f \end{bmatrix}^\dagger \hat{A}^n \begin{bmatrix} \hat{\beta}_0^+ \chi_i \\ \hat{\beta}_0^- \chi_i \end{bmatrix} \right|^2. \end{aligned} \quad (\text{B3})$$

It can also be written in a form, analogous to Eq. (10):

$$\begin{aligned} W_{\chi_i \chi_f} &= \\ &\left(\begin{bmatrix} \chi_f \\ \chi_f \end{bmatrix} \otimes \begin{bmatrix} \hat{\beta}_0^+ \chi_i \\ \hat{\beta}_0^- \chi_i \end{bmatrix} \right)^\dagger \frac{1}{1 - \hat{A} \otimes \hat{A}^\dagger} \begin{bmatrix} \hat{\beta}_0^+ \chi_i \\ \hat{\beta}_0^- \chi_i \end{bmatrix} \otimes \begin{bmatrix} \chi_f \\ \chi_f \end{bmatrix}. \end{aligned} \quad (\text{B4})$$

Though Eq. (B4) is very useful for numerical analysis it yields very cumbersome analytical expressions for \mathcal{T} and \mathbf{P} . For the case $\gamma \ll 1$ it turns out more convenient to use Eq. (B3) for deriving

Eqs. (45)-(48). To this end, we first make a unitary transformation of the matrix \hat{A} :

$$\hat{A}' = \hat{\Lambda} \hat{A} \hat{\Lambda}^{-1}, \quad (\text{B5})$$

described by the block matrix

$$\hat{\Lambda} = \begin{bmatrix} \hat{\Lambda}_+ & 0 \\ 0 & \hat{\Lambda}_- \end{bmatrix}, \quad (\text{B6})$$

where $\hat{\Lambda}_\pm$ are 2×2 matrices which correspond to two unitary transformations diagonalizing matrices \hat{M}_+ and \hat{M}_- , respectively (the transformation, described by Eqs. (B5), (B6), exactly diagonalizes the matrix \hat{A} in the case $t_b = 0$, when \hat{A} is a block matrix with blocks $e^{-2\pi i \phi} \hat{M}_+$ and $e^{2\pi i \phi} \hat{M}_-$). For weak tunneling coupling, $\gamma \ll 1$, and $t_b \neq 0$, the matrix \hat{A}' reads

$$\hat{A}' = \frac{1}{(1 + \gamma)^2} \begin{bmatrix} \lambda_+^2 & 0 & -\gamma c_- \lambda_+ (\lambda_+ + \lambda_-) & \gamma s_- \lambda_+ (\lambda_+ - \bar{\lambda}_-) \\ 0 & \bar{\lambda}_+^2 & -\gamma s_- \bar{\lambda}_+ (\bar{\lambda}_+ - \lambda_-) & -\gamma c_- \bar{\lambda}_+ (\bar{\lambda}_+ + \bar{\lambda}_-) \\ -\gamma c_- \lambda_- (\lambda_- + \lambda_+) & -\gamma s_- \lambda_- (\lambda_- - \bar{\lambda}_+) & \lambda_-^2 & 0 \\ \gamma s_- \bar{\lambda}_- (\bar{\lambda}_- - \lambda_+) & -\gamma c_- \bar{\lambda}_- (\bar{\lambda}_- + \bar{\lambda}_+) & 0 & \bar{\lambda}_-^2 \end{bmatrix}, \quad (\text{B7})$$

where $\lambda_\pm = e^{\mp i \pi (\phi - \delta_\pm)}$, and $\bar{\lambda}_\pm = e^{\mp i \pi (\phi + \delta_\pm)}$. After the unitary transformation, Eq. (B3) is rewritten as follows:

$$W_{\chi_i \chi_f} \approx 4\gamma^2 \sum_{n=0}^{\infty} \left| \begin{bmatrix} \langle \chi_f | \chi_+^1 \rangle \\ \langle \chi_f | \chi_+^2 \rangle \\ \langle \chi_f | \chi_-^1 \rangle \\ \langle \chi_f | \chi_-^2 \rangle \end{bmatrix}^\dagger (\hat{A}')^n \begin{bmatrix} \langle \tilde{\chi}_+^1 | \chi_i \rangle \lambda_+ \\ \langle \tilde{\chi}_+^2 | \chi_i \rangle \bar{\lambda}_+ \\ \langle \tilde{\chi}_-^1 | \chi_i \rangle \lambda_- \\ \langle \tilde{\chi}_-^2 | \chi_i \rangle \bar{\lambda}_- \end{bmatrix} \right|^2. \quad (\text{B8})$$

Here

$$\chi_\pm^1 = \begin{bmatrix} \cos \vartheta_\pm / 2 \\ -\sin \vartheta_\pm / 2 \end{bmatrix}, \quad \chi_\pm^2 = \begin{bmatrix} \sin \vartheta_\pm / 2 \\ \cos \vartheta_\pm / 2 \end{bmatrix}. \quad (\text{B9})$$

are the eigenstates of the spin rotation matrices \hat{M}_+ and \hat{M}_- , respectively, and

$$\tilde{\chi}_\pm^1 = \begin{bmatrix} \cos \vartheta_\pm / 2 \\ \sin \vartheta_\pm / 2 \end{bmatrix}, \quad \tilde{\chi}_\pm^2 = \begin{bmatrix} -\sin \vartheta_\pm / 2 \\ \cos \vartheta_\pm / 2 \end{bmatrix}. \quad (\text{B10})$$

The spinors (B9) and (B10) obey ($\mu = 1, 2$)

$$\begin{aligned} \hat{M}_\pm \chi_\pm^1 &= e^{\pm 2\pi i \delta_\pm} \chi_\pm^1, \\ \hat{M}_\pm \chi_\pm^2 &= e^{\mp 2\pi i \delta_\pm} \chi_\pm^2, \\ \hat{M}_{a \rightarrow b}^\pm \tilde{\chi}_\pm^\mu &= e^{\mp (-1)^\mu \pi i \delta_\pm} \chi_\pm^\mu, \\ \hat{M}_{b \rightarrow a}^\pm \chi_\pm^\mu &= e^{\mp (-1)^\mu \pi i \delta_\pm} \tilde{\chi}_\pm^\mu. \end{aligned} \quad (\text{B11})$$

Since $\gamma \ll 1$, the off-diagonal elements A'_{ij} are small and should be taken into account only when they become comparable with the differences of corresponding diagonal elements: $|A'_{ij}/(A'_{ii} - A'_{jj})| \gtrsim 1$. Thus we can neglect the backscattering for all ϕ , with the exception of the vicinities of the points, where $\lambda_+ = \lambda_-$, $\bar{\lambda}_+ = \bar{\lambda}_-$, $\lambda_+ =$

$-\bar{\lambda}_-$ or $\bar{\lambda}_+ = -\lambda_-$. These equations correspond to the magnetic fluxes $\pm(\delta_+ + \delta_-)/2$ and $\pm(\delta_+ - \delta_-)/2 + 1/2$. Hence, the calculations can be performed in two steps. First, we solve the problem neglecting the backscattering. The obtained solution is valid everywhere except the vicinities of points defined above. Next, we assume that the flux is close to one of these points and take into account backscattering.

The calculation of $W_{\chi_i \chi_f}$ for $t_b = 0$ is straightforward, since A' is diagonal, and leads to the results, given by Eqs. (45)-(48) with the substitutions $\mathcal{T}_0(\phi) \rightarrow \text{Re } F(\phi)$ and $\mathcal{T}_0'(\phi) \rightarrow \text{Im } F(\phi)$, where

$$F(x) = \frac{8\gamma^2 e^{-2\pi i x}}{1 - e^{-4\pi i x}(1 + \gamma)^{-4}} + 2\gamma \quad (\text{B12})$$

Both real and imaginary parts of this function have sharp peaks at $x = 0$ and at $x = 1/2$ with the width on the order of γ . We see that in this approximation $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$ have eight resonances per each period, corresponding to fluxes $\pm(\delta_+ + \delta_-)/2$, $\pm(\delta_+ + \delta_-)/2 + 1/2$, $\pm(\delta_+ - \delta_-)/2$, and $\pm(\delta_+ - \delta_-)/2 + 1/2$. Next we will show that the peaks at $\phi = \pm(\delta_+ + \delta_-)/2$ and $\phi = \pm(\delta_+ - \delta_-)/2 + 1/2$ [which correspond to the peak in $F(x)$ at $x = 0$], disappear, when we take the backscattering into account, so only four resonances in $\mathcal{T}(\phi)$ and $\mathbf{P}(\phi)$ remain. It is worth noting that the latter statement is true only for the interferometer with equal arms (see Ref. 10 for the discussion of the spinless case with unequal arms).

Let us consider, for example, the point $\phi = (\delta_1 + \delta_2)/2$. In the vicinity of this point we can neglect all off-diagonal elements of the matrix \hat{A}' except the elements $-\gamma c \lambda_+(\lambda_+ + \lambda_-)$ and $-\gamma c \lambda_-(\lambda_- + \lambda_+)$, so that the matrix \hat{A}' turns into a block matrix with two 1×1 blocks and a 2×2 block⁴⁹. We can also neglect the interference of the contributions of different blocks when calculating the modulus squared in Eq. (B8), since these interference terms are roughly proportional to $\sum_n (1 + \gamma)^{-4n} \exp(in\Delta\phi)$ with $\Delta\phi \gg \gamma$,⁴⁹ and therefore are small. The points $\phi = -(\delta_1 + \delta_2)/2$ and $\phi = \pm(\delta_+ - \delta_-)/2 + 1/2$ are treated in an analogous way. For each of these points we need to calculate the expression of the following form:

$$\sum_{n=0}^{\infty} \left| \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^\dagger \tilde{A}^n(\phi'; \eta) \begin{bmatrix} \zeta_3 e^{-i\pi\phi'} \\ \zeta_4 e^{i\pi\phi'} \end{bmatrix} \right|^2, \quad (\text{B13})$$

where \tilde{A} is the 2×2 block of the matrix \hat{A}' :

$$\tilde{A}(\phi; \eta) = \frac{1}{(1 + \gamma)^2} \begin{bmatrix} e^{-2\pi i \phi} & -\gamma \eta (e^{-2\pi i \phi} + 1) \\ -\gamma \eta (e^{2\pi i \phi} + 1) & e^{2\pi i \phi} \end{bmatrix}.$$

For example, for the point $\phi = (\delta_1 + \delta_2)/2$, the parameters entering Eq. (B13) are as follows: $\phi' = \phi - (\delta_1 + \delta_2)/2$, $\zeta_1 = \langle \chi_f | \chi_+^1 \rangle$, $\zeta_2 = \langle \chi_f | \chi_-^1 \rangle$, $\zeta_3 = \langle \tilde{\chi}_+^1 | \chi_i \rangle \lambda_+ e^{i\pi\phi'}$, $\zeta_4 = \langle \tilde{\chi}_-^1 | \chi_i \rangle \lambda_- e^{-i\pi\phi'}$, $\eta = c_-$ (note that ζ_3 and ζ_4 do not depend on ϕ).

A convenient way of calculating the expression (B13) is to rewrite it as an integral

$$\int_0^{2\pi} \left| \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}^\dagger (1 - e^{ik} \tilde{A})^{-1} \begin{bmatrix} \zeta_3 e^{-i\pi\phi'} \\ \zeta_4 e^{i\pi\phi'} \end{bmatrix} \right|^2 \frac{dk}{2\pi},$$

make a substitution $z = e^{ik}$ and use the identity

$$(z - \tilde{A})^{-1} = \frac{z - \text{Tr} \tilde{A} + \tilde{A}}{\det(z - \tilde{A})}, \quad (\text{B14})$$

which is valid for any 2×2 matrix. For $\phi' \ll 1$, the calculation yields

$$\frac{1}{8\gamma} \left(|k_1|^2 + \frac{\pi^2 (\phi')^2 |k_3|^2 + \gamma |\eta k_2 - k_1|^2}{\pi^2 (\phi')^2 + \gamma(1 - \eta^2)} \right), \quad (\text{B15})$$

where

$$\begin{aligned} k_1 &= \zeta_1^* \zeta_3 + \zeta_2^* \zeta_4 \\ k_2 &= \zeta_1^* \zeta_4 + \zeta_2^* \zeta_3 \\ k_3 &= \zeta_1^* \zeta_3 - \zeta_2^* \zeta_4 \end{aligned} \quad (\text{B16})$$

Using this expression, we find $W_{\chi_i \chi_f}$ and get the following simple result for the full transmission coefficient and spin polarization in the vicinities of $\phi = \pm(\delta_1 + \delta_2)/2$ and $\phi = \pm(\delta_+ - \delta_-)/2 + 1/2$:

$$\mathcal{T} = 2\gamma, \quad P_x = P_y = P_z = 0. \quad (\text{B17})$$

Thus the backscattering processes destroy the resonances at these points. To write the correct answer, one should replace the function $F(x)$, given by Eq. (B12), with the one that coincides with $F(x)$ for $\gamma \ll x \ll 1 - \gamma$, and is constant in vicinity of $x = 0$ and $x = 1$. Eqs. (13) and (52) represent, respectively, the real and imaginary parts of such a function.

-
- ¹ A. Yacoby, M. Heiblum, V. Umansky, H. Shtrikman, D. Mahalu, Phys. Rev. Lett. **73**, 3149 (1994).
 - ² A. Yacoby, M. Heiblum, D. Mahalu, H. Shtrikman, Phys. Rev. Lett., **74**, 4047 (1994).
 - ³ R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, H. Shtrikman Nature **385**, 417 (1997).
 - ⁴ E. Buks, R. Schuster, M. Heiblum, D. Mahalu, H. Shtrikman, Nature **391**, 871 (1998).
 - ⁵ Y. Aharonov, D. Bohm, Phys. Rev. B **115**, 485 (1959).
 - ⁶ A.G. Aronov, Yu.V. Sharvin, Rev. Mod. Phys. **59**, 755 (1987).
 - ⁷ M. Büttiker, Y. Imry, and M.Ya. Azbel, Phys. Rev. A **30**, 1982 (1984); Y. Gefen, Y. Imry, and M.Ya. Azbel, Phys. Rev. Lett. **52**, 139 (1984); M. Büttiker, Y. Imry, R. Landauer, S. Pinhas, Phys. Rev. B **31**, 6207 (1985).
 - ⁸ J.M. Kinaret, M. Jonson, R.I. Shekhter, S. Eggert, Phys. Rev. B **57**, 3777 (1998).
 - ⁹ E.A. Jagla, C.A. Balseiro, Phys. Rev. Lett. **70**, 639 (1993).
 - ¹⁰ A.P. Dmitriev, I.V. Gornyi, V.Yu. Kachorovskii, D.G. Polyakov Phys. Rev. Lett., **105**, 036402 (2010).
 - ¹¹ Y. Meir, Y. Gefen, O. Entin-Wohlman, Phys. Rev. Lett **63**, 798 (1989).
 - ¹² H. Mathur, A.D. Stone, Phys. Rev. B **44**, 10957 (1991).
 - ¹³ A.G. Aronov, Y.B. Lyanda-Geller, Phys. Rev. Lett **70**, 343 (1993).
 - ¹⁴ T.Z. Qian, Z.B. Su, Phys. Rev. Lett. **72**, 2311 (1994).
 - ¹⁵ J. Nitta, F.E. Meijer, H. Takayanagi, Appl. Phys. Lett **75**, 695 (1999).
 - ¹⁶ D. Frustaglia, K. Richter, Phys. Rev. B **69**, 235310 (2004).
 - ¹⁷ B. Molnar, F.M. Peeters, P. Vasilopoulos, Phys. Rev. B **69**, 155335 (2004).
 - ¹⁸ M.V. Entin and L.I. Magarill, Europhys. Lett. **68** 853 (2004).
 - ¹⁹ V. Gritsev, G. Japaridze, M. Pletyukhov, D. Baeriswyl, Phys. Rev. Lett **94**, 137207 (2005).
 - ²⁰ P. Devillard, A. Crépieux, K.I. Imura, and T. Martin, Phys. Rev. B **72**, 041309 (2005).
 - ²¹ U. Aeberland, K. Wakabayashi, M. Sigrist, Phys. Rev. B **72**, 075328 (2005).
 - ²² M. König, A. Tschetschetkin, E.M. Hankiewicz, J. Sinova, V. Hock, V. Daumer, M. Schäfer, C.R. Becker, H. Buhmann, L.W. Molenkamp, Phys. Rev. Lett. **96**, 076804 (2006).
 - ²³ T. Bergsten, T. Kobayashi, Y. Sekine, J. Nitta, Phys. Rev. Lett. **97**, 196803 (2006).
 - ²⁴ R. Citro, F. Romeo, Phys. Rev. B **73**, 233304 (2006).
 - ²⁵ M. Pletyukhov, V. Gritsev, N. Pauget, Phys. Rev. B **74**, 045301 (2006).
 - ²⁶ R. Citro, F. Romeo, Phys. Rev. B **74**, 115329 (2006).
 - ²⁷ A.A. Kovalev, M.F. Borunda, T. Jungwirth, L.W. Molenkamp, J. Sinova, Phys. Rev. B **76**, 125307 (2007).
 - ²⁸ F. Cheng, G. Zhou, J. Phys.: Condens. Matter **19**, 136215 (2007).
 - ²⁹ F. Romeo, R. Citro, M. Marinaro, Phys. Rev. B **78**, 245309 (2008).
 - ³⁰ A.M. Lobos and A.A. Aligia, Phys. Rev. Lett **100**, 016803 (2008).
 - ³¹ M. Pletyukhov and U. Zülike, Phys. Rev. B **77**, 193304 (2008).
 - ³² V. Moldoveanu and B. Tanatar, Phys. Rev. B **81**, 035326 (2010).
 - ³³ A. Aharony, Y. Tokura, G.Z. Cohen, O. Entin-Wohlman, S. Katsumoto, Phys. Rev. B **84**, 035323 (2011).
 - ³⁴ C.X. Liu, J.C. Budisch, P. Recher, B. Trauzettel, Phys. Rev. B **83**, 035407 (2011).
 - ³⁵ P. Michette and P. Recher, Phys. Rev. B **83**, 125420 (2011).
 - ³⁶ We assume here that E_F is the largest energy scale in the problem, thus replacing in the expression for Ω_Z the energy-dependent electron velocity v with v_F .
 - ³⁷ Y. Aharonov, A. Casher, Phys. Rev. Lett **53**, 319 (1984).
 - ³⁸ H. Mathur, A.D. Stone, Phys. Rev. Lett **68**, 2964 (1992).
 - ³⁹ M.V. Berry, Proc. R. Soc. London A **392**, 45 (1984).
 - ⁴⁰ Beyond the adiabatic limit, the AC phase can be also separated into the dynamical part and the geometrical Aharonov-Anandan phase.⁴¹ The latter is a generalization of the Berry geometrical phase for the nonadiabatic case (see discussion in Ref. 14)
 - ⁴¹ Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).
 - ⁴² S. Lal, S. Rao, and D. Sen, Phys. Rev. B **66**, 165327 (2002).
 - ⁴³ S. Das, S. Rao, and D. Sen, Phys. Rev. B **70**, 085318 (2004).
 - ⁴⁴ D.N. Aristov, A.P. Dmitriev, I.V. Gornyi, V.Yu. Kachorovskii, D.G. Polyakov, and P. Wölffe, Phys. Rev. Lett., **105**, 266404 (2010).
 - ⁴⁵ D.N. Aristov, Phys. Rev. B **83**, 115446 (2011).
 - ⁴⁶ D.N. Aristov and P. Wölffe, Phys. Rev. B **84**, 155426 (2011).
 - ⁴⁷ A.P. Dmitriev, I.V. Gornyi, V.Yu. Kachorovskii, D.G. Polyakov, to be published.
 - ⁴⁸ A.G. Yashenkin, I.V. Gornyi, A.D. Mirlin, and

D.G. Polyakov, Phys. Rev. B **78**, 205407 (2008).

⁴⁹ We restrict our analysis to the case, when the four points $\phi = \pm(r_+ + r_-)/2$ and $\phi =$

$\pm(r_+ - r_-)/2 + 1/2$ are sufficiently far away from each other.